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**RELIABILITY OF STRATIFIED FLOWS OF MICROPOLAR FLUIDS MECHANICS CONCERNS**

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<sup>1</sup>Shubhdeep Verma, <sup>2</sup>Dr.Kapil Kumar Bansal<sup>1</sup>Research Scholar, <sup>2</sup>Supervisor<sup>1,2</sup>Malwanchal UniversityIndore, Madhya Pradesh

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**ABSTRACT**

Fluid mechanics concerns itself with the investigation of motion and equilibrium of fluids. We normally recognize three states of matter: solid, liquid and gas. However, liquid and gas are both fluids: in contrast to solids they lack the ability to resist deformation. Because a fluid cannot resist the deformation force, it moves, it flows under the action of the force. Its shape will change continuously as long as the force is applied. A solid can resist a deformation force while at rest, this force may cause some displacement but the solid does not continue to move indefinitely. Thus, a fluid is a substance that does not has characteristic shape or extensive physical property such as crystalline structure.

**KEY WORDS :** Fluids mechanics, solids, property

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**INTRODUCTION**

Eringen (1964) developed a continuum theory of micro fluid which, roughly speaking, is a fluent medium whose properties and behavior are affected by the local structure and micro motions of the constituent particles contained in each of its volume element. These fluids can support body moments, micro stress averages, stress moments and are influenced by the spin inertia which have no counterpart in the classical fluid theories. Due to the complexities of this theory, its potential has not thus far been exploited. Several sub-classes of this theory have been introduced by Eringen (1969, 1980). Micropolar fluid is one of them which has shown promise for predicting fluid behavior at micro scale. As mention in this paper, compared to the classical Newtonian fluids, micropolar fluids are characterized by two supplementary variables spin and micro-inertia, where the spin is responsible for the micro-rotations and the micro-inertia describes the distribution of atoms and molecules inside the fluid elements in addition to the velocity vector. Micropolar fluids can model ferrofluids, polymers, bubbly liquids, paints, liquid crystals with rigid molecules, clouds with dust, muddy fluids, colloidal fluids, blood and fluids containing certain additives.

**REVIEW OF LITERATURE**

Ariman et al. (2015) presented an excellent review of the study of micropolar fluid and its applications. Batra (2012) considered a heat conducting compressible micropolar fluid at rest and filling a closed stationary rigid container. He showed that the energy of arbitrary disturbances of the fluid eventually decays. Perez-Garcia and Rubi (2006) presented a generalization to micropolar fluids of stability criteria developed by Glansdorff and Prigogine. Perdikis and Reptis (2014) analyzed the steady flow of a micropolar fluid past an unmoving plate by the presence of radiation. Hayakawa (2000) presented a systematic calculation of micropolar fluid flows around a sphere and a cylinder. Stavre (2010) determined an external field when the micro-rotation is equal to the vorticity of the fluid. He studied the discretization of the approximation and proved stability and convergence theorems. Murty (2012) investigated the combined effect of vertical throughflow and magnetic field on an electrically conducting micropolar fluid layer. Ibrahim et al. (2006) analyzed the non-classical heat conduction effects in Stokes' second problem of a micropolar fluid. Aziz (2016) studied the effect of radiation on magneto-hydrodynamic mixed convective steady laminar boundary layer flow of an optically thick electrically conducting viscous micropolar fluid past a moving semi-infinite vertical plate for high

temperature differences.

**FORMULATION OF THE PROBLEM**

Here we consider the stability of an incompressible micropolar fluid confined between infinite horizontal parallel planes at a finite distance  $d$  apart. In the Cartesian frame of reference, the axis of  $x$  is in the main flow direction and the axis of  $z$  is perpendicular to the planes.

The mathematical equations governing the motion of micropolar fluids for the considered flow are

$$\nabla \cdot \mathbf{q} = 0, \tag{1.1}$$

$$\rho \frac{D\mathbf{q}}{Dt} = -\nabla p + (\mu + \mu_r) \nabla^2 \mathbf{q} + \mu_r (\nabla \times \boldsymbol{\omega}) - \rho g \mathbf{e}_z, \tag{1.2}$$

$$\rho_j \frac{D\boldsymbol{\omega}}{Dt} = (\varepsilon' + \beta'') \nabla (\nabla \cdot \boldsymbol{\omega}) + \gamma \nabla^2 \boldsymbol{\omega} + \mu_r (\nabla \times \mathbf{q}) - 2\mu_r \boldsymbol{\omega} \tag{1.3}$$

and  $\frac{D\rho}{Dt} = 0.$  (1.4)

**BASIC STATE**

In the undisturbed state, the fluid is at rest, therefore the basic state is characterized by

$$\mathbf{q} = (0, 0, 0),$$

$$\boldsymbol{\omega} = (0, 0, 0),$$

$$p = p(z)$$

**ANALYTICAL DISCUSSION**

Multiply equation (1.5) by  $w^*$ , the complex conjugate of  $w$ , integrate over the range of  $z$  and make use of boundary conditions, the real and imaginary parts of the resulting equation respectively yield

$$\sigma_i \left[ \int \left( 2\sigma_r l + 2A - \frac{J_\mu k^2}{\sigma_r^2 + \sigma_i^2} \right) (|Dw|^2 + k^2 |w|^2) dz \right. \\ \left. + \int \{1 + l(1 + K)\} (|D^2 w|^2 + 2k^2 |Dw|^2 + k^4 |w|^2) dz - \int \left( \frac{2J_\mu k^2 A}{\sigma_r^2 + \sigma_i^2} \right) (|w|^2) dz \right] = 0 \tag{1.5}$$

In the analysis given below, two cases have been discussed depending upon whether  $J_\square > 0$  or  $J_\square < 0$  i.e. whether the system is statically stable or statically unstable.

**CASE I:  $J_\square > 0$  i.e.  $D\rho_0 < 0$  (STATICALLY STABLE SYSTEM)**

Let  $J_\square > 0$  i.e. the system is statically stable so that the density decreases in the vertically upward direction. In this case following results have been derived

**THEOREM 1:** Non-oscillatory modes are necessarily stable.

**PROOF:** Let the modes be non-oscillatory so that  $\square_i = 0$ , then equation (1.6) yields

$$\int \left( \sigma_r^2 l + 2A\sigma_r + \frac{J_\mu k^2}{\sigma_r} \right) (|Dw|^2 + k^2 |w|^2) dz + \int J_\mu k^2 \left( l + \frac{2A}{\sigma_r} \right) (|w|^2) dz$$

$$+ \int \left[ \sigma_r \{1 + l(1 + K)\} + A(2 + K) \right] (|D^2 w|^2 + 2k^2 |Dw|^2 + k^4 |w|^2) dz \tag{1.6}$$

$$+ \int (1 + K) (|D^3 w|^2 + 3k^2 |D^2 w|^2 + 3k^4 |Dw|^2 + k^6 |w|^2) dz = 0.$$

For the consistency of equation (1.10), we must necessarily have  $\sigma_r < 0$ , which means the stability of non-oscillatory modes. By implication the above result shows that the unstable modes, if exist, will necessarily be oscillatory.

**THEOREM 2:** For  $J_\mu > 0$ , unstable modes ( $\sigma_r > 0$ ), if exist, will lie inside a circle given by

$$\sigma_r^2 + \sigma_i^2 < \frac{J_\mu (k^2 + 2A)}{2A + k^2 \{1 + l(1 + K)\}}.$$

**PROOF:** Let the modes be unstable ( $\sigma_r > 0$ ), then equation (1.7) can be rewritten as

$$\{1 + l(1 + K)\} - \left( \frac{J_\mu (k^2 + 2A)}{\sigma_r^2 + \sigma_i^2} \right) < 0 \tag{1.7}$$

necessarily. Thus the bounds on  $\sigma_r$  and  $\sigma_i$  for arbitrary unstable modes are given by

$$\sigma_r^2 + \sigma_i^2 < \frac{J_\mu (k^2 + 2A)}{2A + k^2 \{1 + l(1 + K)\}}. \tag{1.8}$$

**THEOREM 3:** For  $J_\mu > 0$ , modes will be either neutral and non-oscillatory or oscillatory and non-neutral if

$$J_\mu < \frac{K k^2 (k^2 + A) (2A + l k^2 K)}{l(k^2 + 2A)}. \tag{1.9}$$

**PROOF:** For  $J_\mu > 0$ , let the modes be neutral ( $\sigma_r = 0$ ), and oscillatory ( $\sigma_i \neq 0$ ) then equation (1.10) yields

$$\sigma_i^2 < \frac{J_\mu (k^2 + 2A)}{2A + k^2 \{1 + l(1 + K)\}}. \tag{1.10}$$

Further for neutral modes equation (1.8) provides

Using (1.9), equation (1.10) converts into the following inequality

$$- \int \left( \frac{J_\mu (k^2 + 2A) l}{[2A + k^2 \{1 + l(1 + K)\}]} \right) (|Dw|^2 + k^2 |w|^2) dz + \int J_\mu k^2 l |w|^2 dz$$

$$+ \int A(2 + K) (|D^2 w|^2 + 2k^2 |Dw|^2 + k^4 |w|^2) dz$$

$$+ \int (1 + K) (|D^3 w|^2 + 3k^2 |D^2 w|^2 + 3k^4 |Dw|^2 + k^6 |w|^2) dz < 0.$$

which can be further reduced to

$$\begin{aligned}
 & -\int \left( \frac{J_{\mu} (k^2 + 2A) l}{[2A + k^2 K l]} \right) (|Dw|^2 + k^2 |w|^2) dz + \int J_{\mu} k^2 l |w|^2 dz \\
 & + \int AK (|D^2 w|^2 + 2k^2 |Dw|^2 + k^4 |w|^2) dz \\
 & + \int K (|D^3 w|^2 + 3k^2 |D^2 w|^2 + 3k^4 |Dw|^2 + k^6 |w|^2) dz < 0.
 \end{aligned}$$

or

For the consistency of inequality (1.11) following condition should hold

$$J_{\mu} > \frac{Kk^2 (k^2 + A)(2A + lk^2K)}{l(k^2 + 2A)}. \tag{1.11}$$

Therefore, if

$$J_{\mu} < \frac{Kk^2 (k^2 + A)(2A + lk^2K)}{l(k^2 + 2A)},$$

then either  $\sigma_i = 0$  or  $\sigma_r \neq 0$  i.e. the modes are either non-oscillatory and neutral or if they are oscillatory they will be non-neutral.

**CONCLUSION**

Stability of micropolar fluids between two parallel plates within the framework of linear analysis is examined. Two cases depending upon the statically stable ( $J_{\square} > 0$ ) or statically unstable density stratifications ( $J_{\square} < 0$ ) have been examined both analytically and numerically.

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